To A. E. Douglass
From the Author

AN OPTICAL PERIODOGRAPH

A. E. DOUGLASS

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AN OPTICAL PERIODOGRAPH[1]

By A. E. DOUGLASS

In a preceding article[2] the writer described a photographic device for producing a "periodogram" which promised to have some of the usefulness predicted by its proposer, Professor Arthur Schuster. By its aid the coarser and more persistent fluctuations of value in a series of measures, such as annual sun-spot numbers or rainfall, can be observed with a slight expenditure of mechanical work in plotting, but without mathematical analysis. The experience with that device and with the peculiar variations in the sun-spot numbers strongly indicated the need of more rigid analysis. It became evident indeed that not merely every possible period should be tried as a whole but that every period should be tried in detail and its varying application throughout the entire series of observations made evident and measurable. This need of detailed analysis was in the last article met to a slight extent by a system of multiple plotting in which the solid white curve was repeated many times parallel to itself with regular and equal dark intervals between. Each repetition of the curve also was displaced by a constant amount to one side of its predecessor. Periods then showed themselves by rows of crests which immediately became evident.

[1] Prepared with assistance from the Elizabeth Thompson Science Fund.

[2] *Astrophysical Journal*, **40**, 326, 1914, in which a number of references are given.

Secondary variations showed by lack of straightness in these rows. Any individual row was in effect nothing more than a plot of departures from the constant period represented in the diagram by a vertical line. The duplication of rows assisted the eye in judging of straightness. In the recent improvements of the apparatus this duplicate plotting has been made entirely automatic.

PROCESS

The curve is rendered a solid white area on a black background (or the reverse). The white area showing is limited if necessary by long, straight black mats. To a large camera lens

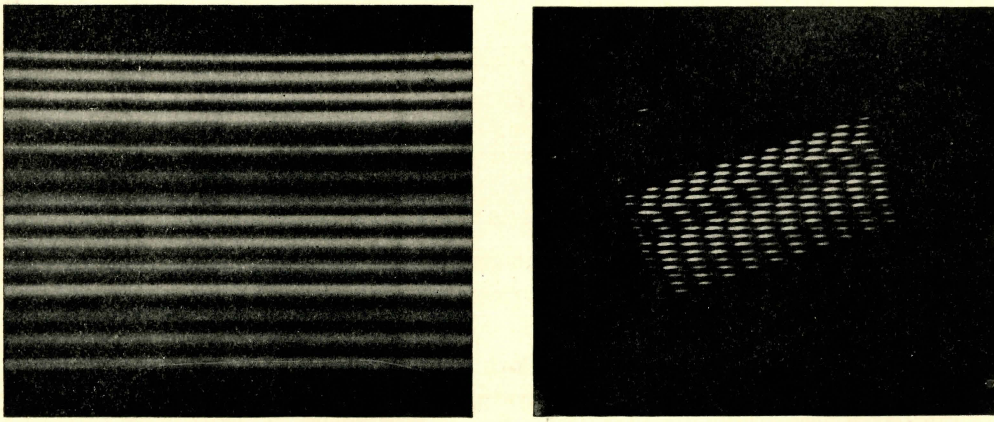

FIG. 1.—"Sweep" of sun-spot numbers and analysis of same by the optical periodograph, showing variations in the period from less than 10 years to nearly 14. Variations are indicated by lack of straightness in the vertical rows.

a cylindric lens is added, by which, in place of a simple image, the curve at the focus is swept or spread across the field, producing a large number of parallel lines, each one proportional in intensity to the height of its corresponding point in the curve (see Fig. 1). Any straight line whatever in any direction across this "sweep" truly represents the original curve, not as a rising and falling line, but in varying light-intensity. A plate with equally spaced parallel opaque lines called the analyzer or analyzing plate is placed in the plane of this "sweep." When the analyzer is turned at a small angle to the lines of the sweep, each transparent line shows the full curve in its varying intensities. These numerous reproductions are all parallel to each other, separated by equal dark lines, and each one is displaced longitudinally with reference to its neighbors (thus presenting the characteristics of the multiple plot). By twisting the analyzer with reference to the sweep, while the two remain in parallel planes, different periods are tested; for as the analyzer twists, each reproduction varies in respect to its length and its displacement from its next adjoining neighbors, above and below. When a period is formed, it shows itself by rows of dark and light spots in alignment more or less perpendicular to the analyzing lines, as in Figs. 1 and 5. These light and dark rows are analogous to interference fringes and are identical with the elaborate but provokingly useless designs on a wire screen in front of its reflection in a window or with the parallel fringes when two sets of parallel lines are held at a slight inclination to each other.[1]

APPARATUS

The curve is now usually reproduced by preparing a narrow strip of tracing-cloth of barely sufficient width to cover its variations. This is pinned over the curve and all area between the curve and one edge of the cloth strip is painted opaque. The strip is then mounted in a narrow horizontal opening in the window and slightly indirect illumination applied so that the translucent parts give out full and even light.

A stand on casters carries the remainder of the apparatus, whose distance from the curve may thus be varied from 3 to 50 feet. A 6-inch stereopticon lens of $1\frac{1}{2}$-inch aperture casts an image of the curve, about 1 inch in length. Between the curve and the lens and almost in contact with the latter the cylindric lens is inserted with axis parallel to the curve. This lens is concave, of 5-inch focus, and cut 1 inch wide in line with the axis and $1\frac{1}{2}$ inches in length at right angles to the axis. The concave side faces the

[1] W. H. Roever has used somewhat similar interference patterns to illustrate very beautifully certain lines of force (*Bulletin of the Mount Weather Observatory*, **6**, 195).

curve. In mounting this lens, about $\frac{1}{8}$ inch is cut off from each dimension.

In the focus of the lens, which may be called the analyzing field, is placed the analyzing plate. It is mounted in the center of a circular disk which rests on wheels and rotates about its center. Four different plates have been made to meet different requirements, the spacing from center to center of the opaque lines being respectively about 0.5, 1.0, 2.0, and 4.0 mm. (It would be well to have intermediate spacings of about 0.7, 1.4, and 2.8 mm in addition.) Considerable difficulty was experienced in making these plates. They are preferably on glass with strong contrast between the opaque and transparent parts. An unexposed dry plate was cleared by a hypo bath and dried. India-ink lines on the film were tried but proved not satisfactory as they were very difficult to produce with perfect regularity and in addition quickly cracked. The grating was finally produced by photographing a 10-foot sheet of co-ordinate paper whose face was covered by line after line (165) of black gummed paper. The co-ordinate lines permitted the spacing to be done with sufficient exactness. The width of the transparent space was throughout three-tenths of the distance from center to center (except in the 4 mm lines when the transparent part is only 0.21; a still smaller space would probably be advantageous). This was carefully photographed by a good lens at three different distances. Two glass prints were made from each negative and then each pair mounted face to face so as

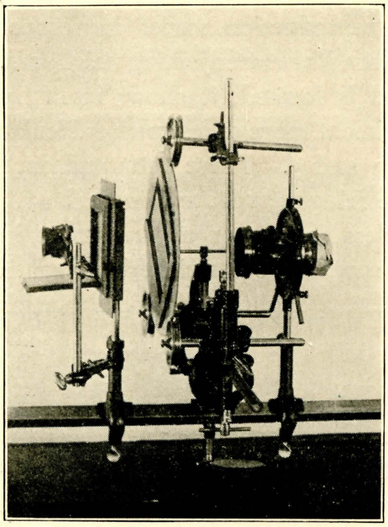

FIG. 2.—Analyzing parts of apparatus with which illustrations in this article were made. From right to left the parts are: cylindric lens, projecting lens, analyzing plate, camera-back and eye-lens. In photographing, the camera-back is moved close to the analyzing plate; in measuring periods, it is replaced by a small position micrometer.

to give great density in the opaque parts. These final plates are 4×5 inches in size and can be interchanged in the rotating disk. Back of the analyzing plate is mounted an eye-lens for visual examination. This lens is about 1 inch in diameter and of 3 inches focus. When a photograph is desired, a camera-back using a $3\frac{1}{4} \times 4\frac{1}{4}$ plate and holder can be put close to the analyzing plate. This arrangement is shown in Fig. 2; the plate-holder, however, in the photograph is moved back a few inches so as not to hide the analyzing plate.

In order to measure the position angle of fringes and obtain other data needed for calculating any period indicated, a small position micrometer can be substituted for the camera-back behind the analyzing field. To get the field reproduced in the plane of the micrometer threads a 3-inch converging lens of 10 or 15 inches focus is placed in or close to the field. This brings the bundle of rays within the compass of a small short-focus lens immediately in front of the micrometer which casts an image of the field in the plane of the threads. Thus the position angle and separation of fringes may be measured, together with the scale of the image of the curve, the spacing of the analyzing lines, and the angle between the analyzing lines and the sweep-lines of the curve. These data are more than are needed to determine the period. Indeed the separation of the fringes may be used as an extra check upon the period found; however, it is not as sensitive as the position angle of the fringe.

Fig. 2 shows the analyzing parts of the periodograph mounted as above described and constructed of ordinary laboratory standards. The only parts at all out of the ordinary are the cylindric lens, which may be obtained at a wholesale optician's, and the analyzing plate.

THEORY

The formula for the period is very simple.

Let y = length of curve in years or other time unit employed
l = length of curve image in cm or other unit of length (across sweep)
s = spacing center to center of analyzing lines in unit of length

Then

$$\frac{l}{s} = \text{number of lines in curve when lines are parallel to sweep}$$

$$\frac{ys}{l} = \text{number of years in 1 line when lines are parallel to sweep}$$

Now taking analyzing lines $\overline{aa'}$ and $\overline{bb'}$ in Fig. 3 as horizontal and letting the sweep be inclined at a small angle δ with the analyzing

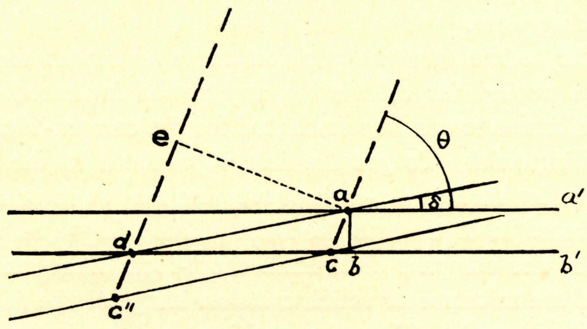

FIG. 3

lines, the number of lines required to cross the sweep in the direction *ab* will be increased and hence the value in years will be decreased; hence

$$\frac{ys}{l} \cos \delta = \text{years per line from } a \text{ to } b$$

If the fringe is perpendicular to the analyzing lines, its period is the distance $\overline{ab}$ in years and we have for this special case:

$$p_1 = \frac{ys}{l} \cos \delta$$

If, however, the fringe takes some other slant as the direction $\overline{ac}$, making the angle θ with the analyzing lines, then the period desired is the time in years between a and c. That equals the time between a and b less the time from b to c. Now $\overline{bc}$ in years would equal $\overline{ab}$ cot θ except for the fact that the horizontal scale along $\overline{bc}$ is greater than the vertical scale along $\overline{ab}$ in the ratio $\frac{\cos \delta}{\sin \delta}$ and therefore a definite space interval along it means fewer years in the ratio of $\frac{\sin \delta}{\cos \delta}$. Hence we have:

$$\overline{bc} \text{ (in years)} = \overline{ab} \text{ (in years)} \tan \delta \cot \theta$$

or

$$P = p_1(1 - \tan \delta \cot \theta)$$

The period thus worked out for Fig. 5*a* is:

$$P = 1.91 - 0.41 \cot \theta$$

and for Fig. 5*b*

$$P = 3.87 - 1.01 \cot \theta$$

The separation of the fringes needs to be known at times in order to find whether one or more actual cycles are appearing in the period under test, as in Fig. 5*b*. In Fig. 3

$$\overline{ab} = s$$

$$\overline{ad} = \frac{s}{\sin \delta}$$

$$\overline{ae} = \frac{s \sin (\theta - \delta)}{\sin \delta}$$

which is the width required.

RANGE OF PERIODS COVERED

There are three ways of testing different periods, namely, twisting the analyzer, changing the scale of the analyzer, and changing the scale of the sweep. The first is a very sensitive method; a large angular change, say 45°, covers a relative range of 100–143. The next size of analyzing plate then carries it from 200 to 286 leaving untested the gap between 143 and 200; hence one sees the advantage of an intermediate size to cover this range. As it was, the gap was covered by moving the lens and the plate on its roller stand which changes the scale of the sweep. Thus twisting the analyzer covers only a short range, but change of scale, whether of the analyzer or of the sweep, may be made to cover any desired range whatever. These two offer excellent checks on each other and should both be used.

SUGGESTIONS FOR PRODUCING A PERIODOGRAM

There appear to be several ways of producing a periodogram, all of which, however, use an additional camera with cylindric lens, a slit in the focal plane, and a plate-holder moving past it by clockwork, after the method described in the preceding article. If the desired range is not great, such prints as in Fig. 5 may be used as the source of light, mounted to rotate by clockwork about a central axis. Or the camera and appurtenances may be mounted in place of the camera-back in Fig. 2 and made to take a photograph as the analyzing plate is turned. The best suggestion, however, is to mount the camera as just described and then take the photograph while changing the scale of the "sweep" through a large range by varying the distance between the mounted curve and the projecting lens. This involves changing the focus of the projecting lens at the same time, which is less simple but not impossible. In this way any desired range would be covered in an instructive and convenient manner. In the author's opinion the most valuable use for the full periodogram will be its aid in presenting results to the reader. Although it may not tell the whole story, yet it covers a large field in a very small space.

A PRACTICAL APPLICATION OF THE PERIODOGRAPH

A period of about 25 months was observed over the eastern and central states by Clayton in 1884–1885[1] in rainfall for the preceding decade. It did not persist and the investigation of it was held in abeyance. Recently a periodic variation in eastern temperatures ranging from 16 to 38 months was studied by Arctowski,[2] who identifies it with the period found by Clayton and entertains the idea that a period may vary. More recently the writer suspected a short period in growth of vegetation both in America and in Northern Europe. It seemed to show in the annual growth of trees in many regions but appeared especially marked in Central Sweden, and only somewhat less so in the Vermont region. The period or "seesaw" indicated by the trees was either 21 or 28 months. In order to distinguish between these alternatives

[1] *American Meteorological Journal*, **1**, 130, 528; and conversation.

[2] *Bulletin of the American Geographical Society*, **45**, 120, 1913.

the rainfall near Windsor, Vermont, was compiled[1] from neighboring records and investigated with the periodograph. All possible periods were tested, but those shown in Fig. 5 were by far the most promising. These diagrams indicate a mean result of about 28 months as the predominating period, but show that this value is not perfectly constant. If this were constant the fringes would be straight. Instead of that the whole fringe is slightly displaced near the center for a considerable period of time. This result of

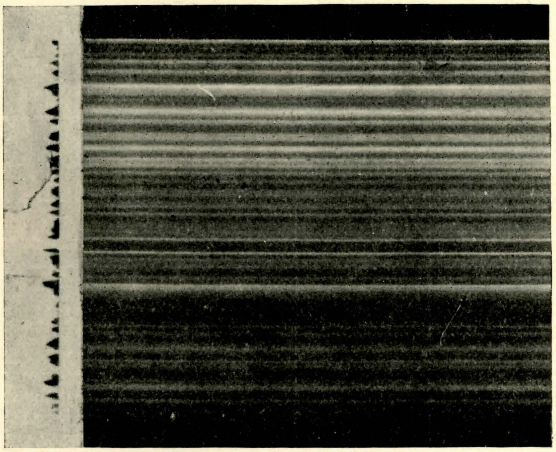

FIG. 4.—Curve of rainfall at Windsor, Vt. (negative), and its "sweep" (positive) from 1835 to 1912.

28 months is believed to be the same as Clayton's result of 25 months, because during the ten years or so preceding 1884 there is a little bend in the fringe which shows that the period was then distinctly less than 28 months, even possibly as low as 25 or 26 months. Arctowski's large allowance of variation easily includes the result here obtained. Hence we find some cause for believing in a period which is not constant. In applying rigid periods this

[1] The curve, shown on a small scale in Fig. 4, was derived as follows: The record of rainfall for every 3 months was smoothed by 4-term overlapping means to get rid of seasonal change. It was then smoothed by Hann's formula (the smoothing here mentioned can be done in the focusing), and "truncated" by a slightly curving line which eliminated coarse variations of ten years or more, leaving the smaller variations. Fig. 5*d* was made directly from this curve. Figs. 5*a*, *b*, and *c* were photographed from a dense photographic negative of the curve.

is likely to be overlooked. The possibility inferred from these photographic results then is that the study of slightly variable periods might open up added fields of knowledge of meteorological phenomena. To the investigation of such periods, the instrument here described is especially adapted.

Inverted →

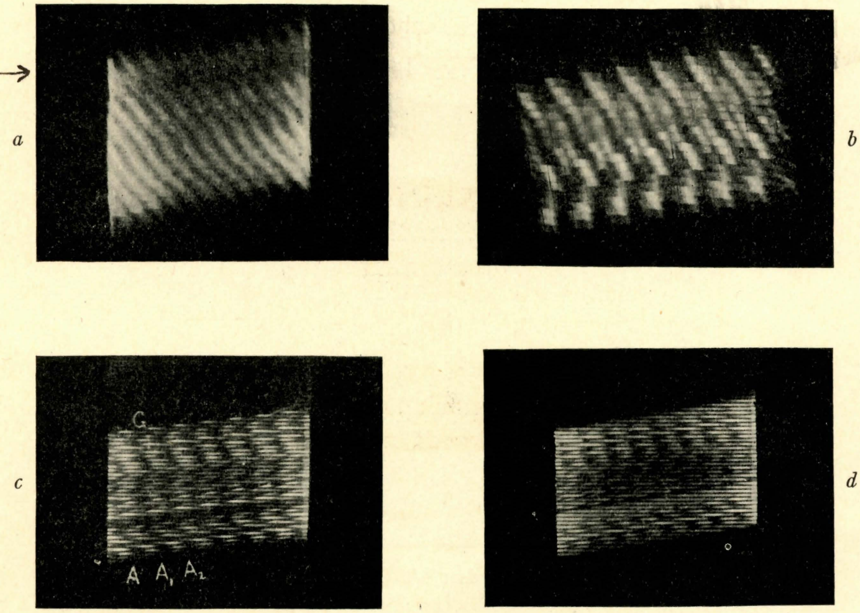

FIG. 5.—Various analyses of rainfall at Windsor, Vt., by the periodograph, showing variation of a suspected 2.3 year period (1912 at top, 1835 at bottom of each). *a*, out of focus, 2.3 year fringes; *b*, out of focus, tested for a 5-year period, it again shows 2.3 year fringes; *c*, in focus, with slight variations of scale and angle from *a;* taken from white curve on black background; *d*, taken from black curve on a white background.

APPLICATION TO THE STUDY OF TREE GROWTH

Fig. 6 shows the application of the periodograph to the analysis of a tree-growth curve. The 500-year record of the yellow pine in northern Arizona, derived from the annual rings, was corrected for decrease of growth with age, reduced to percentages of its own mean, smoothed by Hann's formula, slightly "truncated" to eliminate long fluctuations of over 20 years, and then treated as described above for the rainfall-curves. The result is shown in the figure. A period varying from 5.1 to 6.3 years seems to be traceable throughout. Taken from end to end it averages 5.7 years. In the later centuries it is evidently a submultiple of an approximate 21-year period. Between 1750 and 1790 a period of 4.3 is shown in this analysis, and from 1860 to 1900 a 7-year period is manifest.

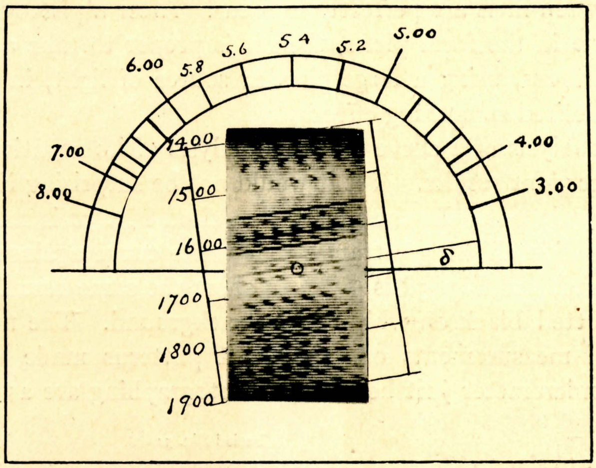

FIG. 6.—Analysis of 500 years of tree-growth record in Arizona to show periods between 3 and 8 years; dates are given on the left; the numbers on the semicircle at the top give the length in years of any period denoted by a fringe extending in that direction.

The vertical fringes suggest a period varying between 5.1 and 6.3 years, best developed between A.D. 1400 and A.D. 1650.

MANIPULATION; ACCURACY

While the optical periodograph has worked satisfactorily in the cases already tested, it has not been free from difficulties. The matter of focus is important. Displacement of the photographic plate a centimeter or two back of the analyzing plane produces a deceptive certainty of result, for the effect of the cylindric lens is to extend the vertical and diminish horizontal alignments. This may be seen in the difference between Figs. 5*a* and 5*c* since the two were taken under nearly the same conditions, but the former is somewhat out of focus. Nevertheless, the actual periods measured in Figs. 5*a* and 5*b* agree within 2 per cent with those in Figs. 5*c* and 5*d*.

Again, for consistent results the analyzing plate must be close to the focal plane of the sweep. If it is not, then distortion is produced in the micrometer field. In order to bring it in the proper place a microscope with a 1″ or 2″ objective is focused on the analyzing lines while the projection lens is moved back and forth till the sweep-lines are perfectly in focus. After a photograph has been made in the focal plane, then it is proper to take a print of it out of focus, using a large circular source of light, in order to get any desired smoothing effect.

The analysis of a curve and the analysis of its negative should give concordant results. Yet it would not be surprising if in some cases they do not. In the illustration cited Fig. 5*c* is made from a lantern slide showing a transparent white curve against a black background, and Fig. 5*d* is given to show the same results with a curve plotted black on white paper background. The numerical results of measurements on equivalent patterns made with the position micrometer just before the photographing are as follows:

	Period Fig. 5*c*	Period Fig. 5*d*
About 1851 to about 1881	2.39 years	2.34 years
" 1881 " " 1887	2.14 "	2.13 "
" 1887 " " 1902	2.46 "	2.38 "
Mean Period through Entire Series 1835–1912		
A to G	2.303 years	2.274 years
A_1 " G	2.342 "	2.360 "
A_2 " G	2.505 "	2.460 "

These differences, amounting to between 1 and 2 per cent, are doubtless due to different estimates of the direction of fringes.[1]

In changing from a curve expressed in ordinates to one expressed in light-intensities, there is no great drawback for this purpose, although precision is lost. The chief loss is in accuracy of photographic values, for relative intensities will vary slightly with the plate, the exposure, and the time and character of the development.

[1] The ideal condition will perhaps use neither positive nor negative type of curve but a plot, on a neutral tone or gray, of departu res from a mean value, positive differences being white, and negative being black.

The illustrations here given and perhaps usually given will tell only a part of the story. It would require a motion picture to tell the whole. The operator at the instrument can see the whole, for it unfolds itself before him as the size of the sweep starting at a minimum increases several hundred per cent. In picking out individual positions for photographs, his own judgment, or personal equation, or prejudice even may enter. But it is a question if they can do material harm, for it seems to the writer almost impossible to estimate offhand what period is being indicated by any fringe on account of the different scales and angles entering every new combination. However, in addition to special photographs, others taken at regular intervals throughout the entire range would give confidence to the reader. A periodogram, though helping very materially, is not completely satisfactory because it may fail to show some promising periods which are not perfectly constant. It is probable that a periodogram of Fig. 5*a* would largely lose the periodic effect there shown.

Again, there may be much uncertainty and perhaps even mistake in judging of the alignments, but the reader to whom the pictures are presented can judge for himself. In fact, the real advantage in this method of analysis is that alternative alignments indicate real uncertainties in the solution, which the reader himself is perfectly able to see and estimate.

In studying diagrams, fringes are emphasized by viewing the print in a slanting direction.

No practical attempt has yet been made to analyze a curve full of sharp variations such as an unsmoothed rainfall curve. A slight variation in focus may be used to smooth such irregularities.

CONCLUSION

In answer to the need of considering secondary variations in any suspected periodic variations, an instrument has been constructed by which, in an hour or two after plotting, the investigator may view all possible periods in a curve and their variations and begin measuring and photographing the promising ones. This optical periodograph is not yet complete, but it is believed that even now it will have some use in preliminary tests of periodic variation and in some cases save long computations or in others show where rigorous investigation should be applied. Should it prove desirable to enter upon extended studies of slightly varying periods, this instrument gives a rapid method of approaching the work.

University of Arizona
February 5, 1915